Analyse & Approaches

1 Page Formula Sheet - IB Mathematics SL & HL

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Prior learning – SL & HL

Area: parallelogram	$A = bh$, b=base, h=height
Area: triangle	$A = \frac{1}{2}(bh)$, b=base, h=height
Area: trapezoid	$A = \frac{1}{2}(a+b)h$, a, b =parallel sides, h= height
Area: circle	$A = \pi r^2$, r = radius
Circumference circle	$C = 2\pi r$, r = radius
Volume: cuboid	$V = lwh$, l =length, w=width, h= height
Volume: cylinder	$V = \pi r^2 h$, r=radius, h=height
Volume: prism	$V = Ah$, A= cross-section area, h=height
Area: cylinder curve	$A = 2\pi rh$, r=radius, h=height
Distance between two points $(x_1, y_1), (x_2, y_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of midpoint	$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

Topic 1: Number and algebra – SL & HL

SL 1.2	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms	$S_n = \frac{n}{2}(2u_1 + (n-1)d = \frac{n}{2}(u_1 + u_n)$
SL 1.3	The n^{th} term of a geometric sequence	$u_n = u_1 r^{n-1}$
	The sum of n terms	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}, r \neq 1$
SL 1.4	Compound interest	$FV = PV \times (1 + \frac{r}{100k})^{kn}$ FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, r% is the nominal annual rate of interest
SL 1.5	Exponents & logarithms	$a^x = b \Leftrightarrow x = \log_a b$, $a > 0$, $b > 0$, $a \neq 1$
SL 1.7	Exponents and logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = \text{m} \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
SL 1.8	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}, \lvert r \rvert < 1$
SL 1.9	Binomial theorem	$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n$ $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

Topic 1: Number and algebra – HL only

AHL 1.10	Combinations	${}^nC_r = \frac{n!}{r!(n-r)!}$
	Permutations	${}^nP_r = \frac{n!}{(n-r)!}$
AHL 1.12	Complex numbers	$z = a + bi$
AHL 1.13	Modulus-argument (polar) & exponential (Euler) form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta} = rcis$
AHL 1.14	De Moivre's theorem	$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta} = r^n cisn\theta$

Topic 2: Functions – SL & HL

SL 2.1	Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
SL 2.6	Axis of symmetry of a quadratic function	$f(x) = ax^2 + bx + c \rightarrow x = \frac{-b}{2a}$
SL 2.7	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$
SL 2.9	Exponential & logarithmic functions	$a^x = e^{x \ln a}$; $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$

Topic 2: Functions – HL only

AHL 2.12	Sum and product of the roots of polynomial equations of the form	$\sum_{r=0}^{n} a_r x^r = 0 \Rightarrow$ Sum is $\frac{-a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$

Topic 3: Geometry and trigonometry – SL & HL

SL 3.1	Distance between two points (x_1, y_1, z_1) & (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
	Coordinates of the midpoint of a line segment	$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}; \frac{z_1 + z_2}{2}\right)$
	Volume: right-pyramid	$V = \frac{1}{3}Ah$, A = base area, h = heigh
	Volume: right cone	$V = \frac{1}{3}\pi r^2 h$, r= radius, h = height
	Area: cone	$A = \pi rl$, r= radius, l = slant height
	Volume: sphere	$V = \frac{4}{3}\pi r^3$, r = radius
	Surface: sphere	$A = 4\pi r^2$, r = radius
SL 3.2	Sine rule	$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$
	Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Area of a triangle	$A = \frac{1}{2}ab\sin C$
SL 3.4	Length of an arc	$l = r\theta$; θ = angle in radians, r = radius
	Area of a sector	$A = \frac{1}{2} \times r^2\theta$ θ = angle in radians, r = radius
SL 3.5	Identity for tanθ	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
SL 3.6	Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
	Double angle identities	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = cos^2\theta - sin^2\theta$ $= 2cos^2\theta - 1 = 1 - 2sin^2\theta$

Topic 3: Geometry and trigonometry – HL only

AHL 3.9	Reciprocal trigonometric identities	$sec\,\theta = \frac{1}{cos\,\theta}$; $cosec\,\theta = \frac{1}{sin\,\theta}$
	Pythagorean identities	$1 + tan^2\theta = sec^2\theta$; $1 + cot^2\theta = cosec^2\theta$
AHL 3.10	Compound angle identities	$\sin(A \pm B) = sinAcosB \pm cosAsinB$ $\cos(A \pm B) = cosAcosB \mp sinAsinB$ $\tan(A \pm B) = \frac{tanA \pm tanB}{1 \mp tanAtanB}$
	Double angle identity for tan	$tan2\theta = \frac{2tan\theta}{1 - tan2\theta}$
AHL 3.12	Magnitude of a vector	$\lvert v \rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
AHL 3.13	Scalar product	$v \cdot w = v_1w_1 + v_2w_2 + v_3w_3$ $v \cdot w = \lvert v \rvert \lvert w \rvert \cos\theta$ θ: angle between v and w
	Angle between two vectors	$\cos\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{\lvert v \rvert \lvert w \rvert}$
AHL 3.14	Vector equ. of a line	$r = a + \lambda b$
	Parametric form of the Equ. of a line	$x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$
	Cartesian equations of a line	$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$
AHL 3.16	Vector product	$v \times w = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$ $\lvert v \times w \rvert = \lvert v \rvert \lvert w \rvert \sin\theta$ θ: angle between v and w
	Area of a parallelogram	$A = \lvert v \times w \rvert$, v and w form two adjacent sides of a parallelogram
AHL 3.17	Vector equ. of a plane	$r = a + \lambda b + \mu c$
	Equ. of a plane	$r \cdot n = a \cdot n$ (using the normal vector)
	Cartesian equ. of a plane	$ax + by + cz = d$

Topic 4: Statistics and probability – SL & HL

SL 4.2	Interquartile range	$IQR = Q_3 - Q_1$
SL 4.3	Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$, where $n = \sum_{i=1}^{k} f_i$
SL 4.5	Probability of an event A	$P(A) = \frac{n(A)}{n(u)}$
	Complementary events	$P(A) + P(A') = 1$
SL 4.6	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A\lvert B) = \frac{P(A \cap B)}{P(B)}$
	Independent events	$P(A \cap B) = P(A)P(B)$
SL 4.7	Expected value of a discrete random variable X	$E(X) = \sum x P(X = x)$
SL 4.8	Binomial distribution Mean -Variance	$X \sim B(n, p)$ $E(X) = np$; $Var(X) = np(1-p)$
SL 4.12	Standardized normal variable	$z = \frac{x - \mu}{\sigma}$

Topic 4: Statistics and probability – HL only

AHL 4.13	Bayes' theorem	$P(B\lvert A) = \frac{P(B)P(A\lvert B)}{P(B)P(A\lvert B) + P(B')P(A\lvert B')}$ $P(B_i\lvert A) = \frac{P(B_i)P(A\lvert B_i)}{P(B_1)P(A\lvert B_1) + P(B_2)P(A\lvert B_2) + P(B_3)P(A\lvert B_3)}$
AHL 4.14	Variance σ^2	$\sigma^2 = \frac{\sum_{i=1}^{k} f_i(x_i - \mu)^2}{n} = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$
	Standard deviation σ	$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i(x_i - \mu)^2}{n}}$
	Linear transformation of a single random variable	$E(aX + b) = aE(X) + b$ $Var(aX + b) = a^2Var(X)$
	Expected value of a continuous random variable X	$E(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx$
	Variance	$Var(X) = E(X - \mu)^2 = E(X^2) - [E(X)]^2$
	Variance of a discrete random variable X	$Var(X) = \sum(x - \mu)^2 P(X = x)$ $= \sum x^2 P(X = x) - \mu^2$
	Variance of a continuous random variable X	$Var(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$ $= \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$

Topic 5: Calculus – SL & HL

SL 5.3	Derivative of x^n	$f(x) = x^n \rightarrow f'(x) = nx^{n-1}$
SL 5.5	Integral of x^n	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
	Area of region enclosed by a curve $y = f(x)$ and the x-axis	$A = \int_a^b y\,dx$, where $f(x) > 0$
SL 5.6	Derivative of $\sin x$	$f(x) = \sin x \rightarrow f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \rightarrow f'(x) = -\sin x$
	Derivative of $\tan x$	$f(x) = \tan x \rightarrow f'(x) = \frac{1}{\cos^2 x}$
	Derivative of e^x	$f(x) = e^x \rightarrow f'(x) = e^x$
	Derivative of $\ln x$	$f(x) = \ln x \rightarrow f'(x) = \frac{1}{x}$
	Chain rule	$y = g(u), u = f(x) \rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
	Product rule	$y = uv \rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
	Quotient rule	$y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
SL 5.9	Acceleration	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$
	Distance ; Displacement travelled from t_1 to t_2	$dist = \int_{t_1}^{t_2} \lvert v(t) \rvert dt$; $disp = \int_{t_1}^{t_2} v(t)\,dt$
SL 5.10	Standard integrals	$\int \frac{1}{x} dx = \ln\lvert x \rvert + C$ $\int \sin x\,dx = -\cos x + C$ $\int \cos x\,dx = \sin x + C$ $\int \frac{1}{\cos^2 x} dx = \tan x + C$ $\int e^x dx = e^x + C$
SL 5.11	Area of region enclosed by a curve and x-axis	$A = \int_a^b \lvert y \rvert\,dx$

Topic 5: Calculus – HL only

AHL 5.12	Derivative of $f(x)$ from first principles	$\frac{dy}{dx} = f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$
AHL 5.15	Standard derivatives	$f(x) = \tan x \Rightarrow f'(x) = sec^2x$ $f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$ $f(x) = cosec\,x \Rightarrow f'(x) = -cosec\,x \cot x$ $f(x) = \cot x \Rightarrow f'(x) = -cosec^2 x$ $f(x) = a^x \Rightarrow f'(x) = a^x(\ln a)$ $f(x) = log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$ $f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1 - x^2}}$ $f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1 - x^2}}$ $f(x) = arctan\,x \Rightarrow f'(x) = \frac{1}{1 + x^2}$
AHL 5.15	Standard integrals	$\int a^x dx = \frac{1}{\ln a}a^x + C$ $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = arcsin\left(\frac{x}{a}\right) + C, \lvert x \rvert < a$
AHL 5.16	Integration by parts	$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$
AHL 5.17	Area of region enclosed by a curve and y-axis	$A = \int_a^b \lvert x \rvert\,dy$
	Volume of revolution about the x or y-axes	$V = \int_a^b \pi y^2\,dx$ or $V = \int_a^b \pi x^2\,dy$
AHL 5.18	Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n)$; $x_{n+1} = x_n + h$ where h is a constant (step length)
	Integrating factor for $y' + P(x)y = Q(x)$	$e^{\int P(x)dx}$
AHL 5.19	Maclaurin series	$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots$
	Maclaurin series for special functions	$e^x = 1 + x + \frac{x^2}{2!} + \cdots$ $ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$ $arctan\,x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$