Applications & Interpretation

1 Page Formula Sheet IB Mathematics SL & HL

First examinations 2021

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Prior learning – SL & HL	
Area: parallelogram	$A = bh$, b=base, h=height
Area: triangle	$A = \frac{1}{2}(bh)$, b=base, h=height
Area: trapezoid	$A = \frac{1}{2}(a+b)h$, a, b =parallel sides, h= height
Area: circle	$A = \pi r^2$, r = radius
Circumference circle	$C = 2\pi r$, r = radius
Volume: cuboid	$V = lwh$, l =length, w=width, h= height
Volume: cylinder	$V = \pi r^2 h$, r=radius, h=height
Volume: prism	$V = Ah$, A= cross-section area, h=height
Area: cylinder curve	$A = 2\pi\, rh$, r=radius, h=height
Distance between two points $(x_1, y_1), (x_2, y_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of midpoint	$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

Prior learning – HL only	
Solutions of a quadratic equation	$ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$

	Topic 1: Number and algebra – SL & HL	
SL 1.2	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms	$S_n = \frac{n}{2}(2u_1 + (n-1)d = \frac{n}{2}(u_1 + u_n)$
SL 1.3	The n^{th} term of a geometric sequence	$u_n = u_1\, r^{n-1}$
	The sum of n terms	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}, r \neq 1$
SL 1.4	Compound interest	$FV = PV \times (1 + \frac{r}{100k})^{kn}$ FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, r% is the nominal annual rate of interest
SL 1.5	Exponents & logarithms	$a^x = b \Leftrightarrow x = log_a b, a > 0, b > 0, a \neq 1$
SL 1.6	Percentage error	$\varepsilon = \left\lvert\frac{v_a - v_e}{v_e}\right\rvert \times 100\%$ v_e =the exact value and v_a=the approximate value

	Topic 1: Number and algebra – HL only	
AHL 1.9	Laws of logarithms	$log_a xy = log_a x + log_a y$ $log_a \frac{x}{y} = log_a x - log_a y$ $log_a x^m = m\, log_a x$ For $x, y, a > 0$
AHL 1.11	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}, \lvert r\rvert < 1$
AHL 1.12	Complex numbers Discriminant	$z = a + bi$ $\Delta = b^2 - 4ac$
AHL 1.13	Modulus-argument (polar) & exponential (Euler) form	$z = r(cos\,\theta + i\,sin\,\theta) = re^{i\theta} = rcis$
AHL 1.14	Determinant of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow det\, A = \lvert A\rvert = ad - bc$
	Inverse of a 2×2 matrix	$A^{-1} = \frac{1}{det\, A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
AHL 1.15	Power formula for a matrix	$M^n = PD^nP^{-1}$, where P is the matrix of eigenvectors and D is the diagonal matrix of eigenvalues

	Topic 2: Functions – SL & HL	
SL 2.1	Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
SL 2.5	Axis of symmetry of a quadratic function	$f(x) = ax^2 + bx + c \rightarrow x = \frac{-b}{2a}$

	Topic 2: Functions – HL only	
AHL 2.9	Logistic function	$f(x) = \frac{L}{1 + Ce^{-kx}}, L, k, C > 0$

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	Topic 3: Geometry and trigonometry – SL & HL	
SL 3.1	Distance between two points (x_1, y_1, z_1) & (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
	Coordinates of the midpoint of a line segment	$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}; \frac{z_1 + z_2}{2}\right)$
	Volume: right-pyramid	$V = \frac{1}{3}Ah$, A = base area, h = heigh
	Volume: right cone	$V = \frac{1}{3}\pi r^2 h$, r= radius, h = height
	Area: cone	$A = \pi rl$, r= radius, l = slant height
	Volume: sphere	$V = \frac{4}{3}\pi r^3$, r = radius
	Surface: sphere	$A = 4\pi r^2$, r = radius
SL 3.2	Sine rule	$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$
	Cosine rule	$c^2 = a^2 + b^2 - 2ab\, cos\, C$ $cos\, C = \frac{a^2 + b^2 - c^2}{2ab}$
	Area of a triangle	$A = \frac{1}{2}ab\, sin\, C$
SL 3.4	Length of an arc	$l = \frac{\theta}{360} \times 2\pi r$; θ = angle in degrees, r = radius
	Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$ θ = angle in degrees, r = radius

	Topic 3: Geometry and trigonometry – HL only	
AHL 3.7	Length of an arc	$l = r\theta$; r = radius, θ = angle in radians
	Area of a sector	$A = \frac{1}{2}r^2\theta$
AHL 3.8	Identities	$Cos^2\,\theta + sin^2\,\theta = 1$ $tan\,\theta = \frac{sin\theta}{cos\theta}$
AHL 3.9	Transformation matrices	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ reflection in the line $y = (\tan\theta)x$ $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ horizontal stretch by scale factor of k $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$ vertical stretch with scale factor of k $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ centre (0,0) enlargement with scale factor of k $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, anticlockwise rotation of angle θ about the origin ($\theta > 0$) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, clockwise rotation of angle θ about the origin ($\theta > 0$)
AHL 3.10	Magnitude of a vector	$\lvert v\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
AHL 3.11	Vector equ. of a line	$r = a + \lambda b$
	Parametric form of the Equ. of a line	$x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$
AHL 3.13	Scalar product	$v \cdot w = v_1w_1 + v_2w_2 + v_3w_3$ $v \cdot w = \lvert v\rvert\lvert w\rvert\, cos\,\theta$ θ: angle between v and w
	Angle between two vectors	$cos\,\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{\lvert v\rvert\lvert w\rvert}$
	Vector product	$v \times w = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$ $\lvert v \times w\rvert = \lvert v\rvert\lvert w\rvert\, sin\,\theta$ θ: angle between v and w
	Area of a parallelogram	$A = \lvert v \times w\rvert$, v and w form two adjacent sides of a parallelogram

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	Topic 4: Statistics and probability – SL & HL	
SL 4.2	Interquartile range	$IQR = Q_3 - Q_1$
SL 4.3	Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$, where $n = \sum_{i=1}^{k} f_i$
SL 4.5	Probability of an event A	$P(A) = \frac{n(A)}{n(u)}$
	Complementary events	$P(A) + P(A') = 1$
SL 4.6	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A\lvert B) = \frac{P(A \cap B)}{P(B)}$
	Independent events	$P(A \cap B) = P(A)P(B)$
SL 4.7	Expected value of a discrete random variable X	$E(X) = \sum x\, P(X = x)$
SL 4.8	Binomial distribution Mean -Variance	$X \sim B(n, p)$ $E(X) = np$; $Var(X) = np(1-p)$

	Topic 4: Statistics and probability – HL only	
AHL 4.14	Linear transformation of a single random variable	$E(aX + b = aE(X) + b$ $Var(aX + = a^2Var(X)$
	Linear combinations of n independent random variables, $X_1, X_2, \ldots, X_n$	$E(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n)$ $= a_1E(X_1) \pm a_2E(X_2) \pm \ldots \pm a_nE(X_n)$ $Var(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n)$ $= a_1^2Var(X_1) + a_2^2Var(X_2) + \cdots + a_n^2Var(X_n)$
	Unbiased estimate of population variance	$s_{n-1}^2 = \frac{n}{n-1}\, s_n^2$ Sample statistics
AHL 4.17	Poisson distribution Mean Variance	$X \sim Po(m)$ $E(X) = m$; $Var(X) = m$
AHL 4.19	Transition matrices	$T^n s_0 = s_n$, where s_0 is the initial state

	Topic 5: Calculus – SL & HL	
SL 5.3	Derivative of x^n	$f(x) = x^n \rightarrow f'(x) = nx^{n-1}$
SL 5.5	Integral of x^n	$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
	Area of region enclosed by a curve $y = f(x)$ and the x-axis	$A = \int_a^b y\, dx$, where $f(x) > 0$
SL 5.8	The trapezoidal rule	$\int_a^b y\, dx \approx \frac{1}{2}h((y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1}))$

	Topic 5: Calculus – HL only	
AHL 5.9	Derivative of $sin\, x$	$f(x) = sin\, x \rightarrow f'(x) = cos\, x$
	Derivative of $cos x$	$f(x) = cos\, x \rightarrow f'(x) = -sin\, x$
	Derivative of $tan\, x$	$f(x) = tan\, x \rightarrow f'(x) = \frac{1}{cos^2\, x}$
	Derivative of e^x	$f(x) = e^x \rightarrow f'(x) = e^x$
	Derivative of $ln\, x$	$f(x) = ln\, x \rightarrow f'(x) = \frac{1}{x}$
	Chain rule	$y = g(u), u = f(x) \rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
	Product rule	$y = uv \rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
	Quotient rule	$y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
AHL 5.11	Standard integrals	$\int \frac{1}{x}\, dx = ln\lvert x\rvert + C$ $\int sin\, x\, dx = -cos\, x + C$ $\int cos\, x\, dx = sin\, x + C$ $\int \frac{1}{cos^2 x}\, dx = tan\, x + C$ $\int e^x\, dx = e^x + C$
AHL 5.12	Area of region enclosed by a curve and x or y-axes	$A = \int_a^b \lvert y\rvert\, dx$ or $A = \int_a^b \lvert x\rvert\, dy$
	Volume of revolution about x or y-axes	$V = \int_a^b \pi y^2\, dx$ or $V = \int_a^b \pi x^2\, dy$
AHL 5.13	Acceleration	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$
	Distance travelled from t_1 to t_2	$dist = \int_{t_1}^{t_2} \lvert v(t)\rvert\, dt$
	Displacement from t_1 to t_2	$disp = \int_{t_1}^{t_2} v(t)\, dt$
AHL 5.16	Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n)$; $x_{n+1} = x_n + h$ where h is a constant (step length)
	Euler's method for coupled systems	$x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n)$ $y_{n+1} = y_n + h \times f_2(x_n, y_n, t_n)$ $t_{n+1} = t_n + h$ where h is a constant (step length)
AHL 5.17	Exact solution for coupled linear differential equations	$x = Ae^{\lambda_1 t}p_1 + Be^{\lambda_2 t}p_2$